Contents

- Extreme value statistics
- Wind and loads
- Example
- Realization in *Gumbelwind*

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Extreme value theory

Aim: prediction of extreme events

Dilemma: extreme events = extremly rare

⬇

lack of data

Resource: limit theorems

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Extreme value theory

Limit theorem 1: The distribution of block extremal values approaches the following distribution function for growing block size

$$H(x) = \begin{cases} e^{-(1+\xi \frac{x-\mu}{\sigma})^{\frac{1}{\xi}}} & \text{für} \quad \xi \neq 0 \\ e^{-e^{-\frac{x-\mu}{\sigma}}} & \text{für} \quad \xi = 0 \end{cases}$$

Fischer/Tippet 1928, generalized extreme value distribution

representation due to Jenkinson/von Mises

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Extreme value theory

The shape parameter ξ determines the type of extreme value distribution and therefore the tail behaviour.

$\xi=0$: Type I – Gumbel distibution

$\xi>0$: Type II – Frechet distribution

$\xi<0$: Typ III – Weibull distribution

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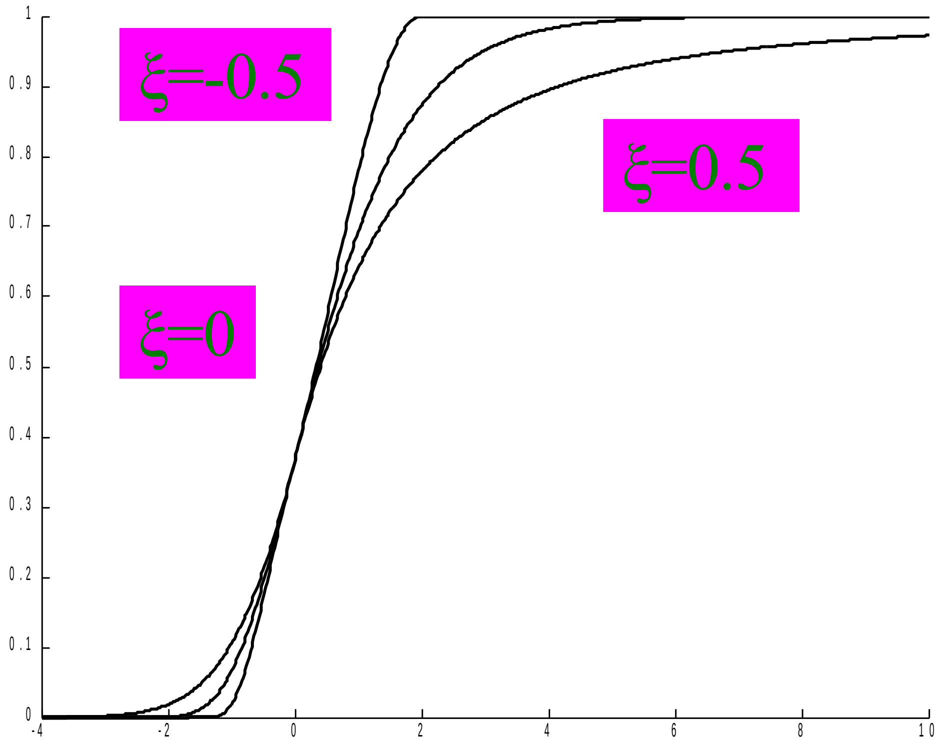
Extreme value theory

Distribution function H(x)



$\mu=0$

$\sigma=1$

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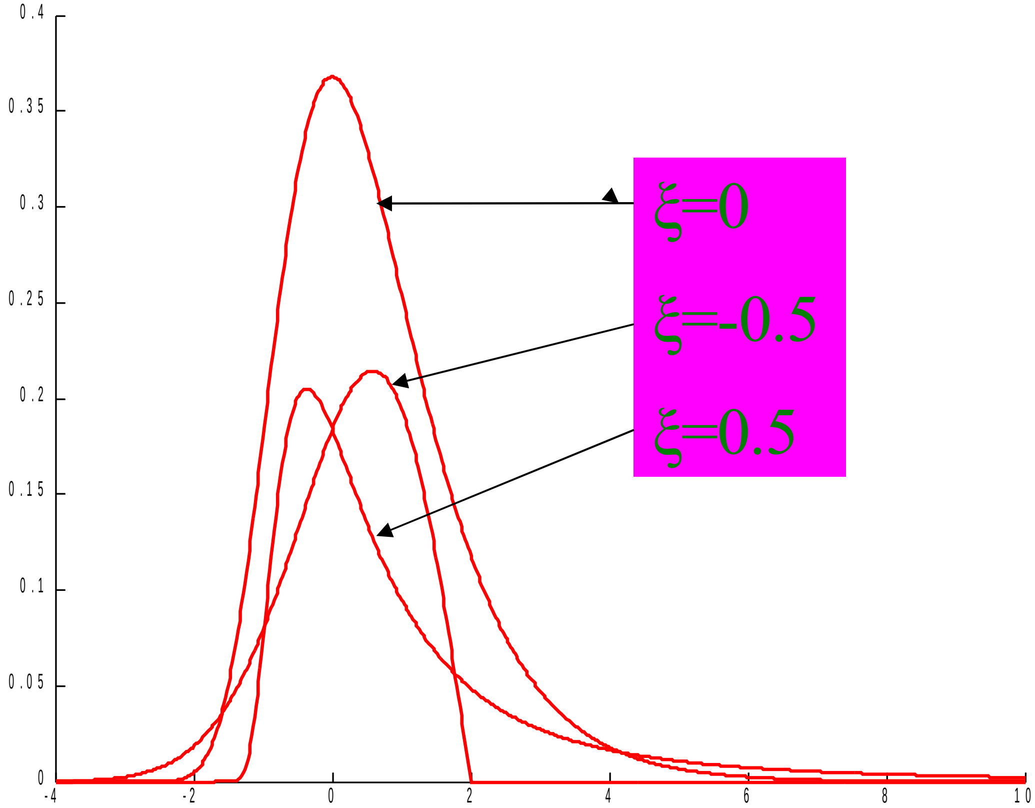
Extreme value theory

probability density h(x)



μ=0

σ=1

Extreme value theory

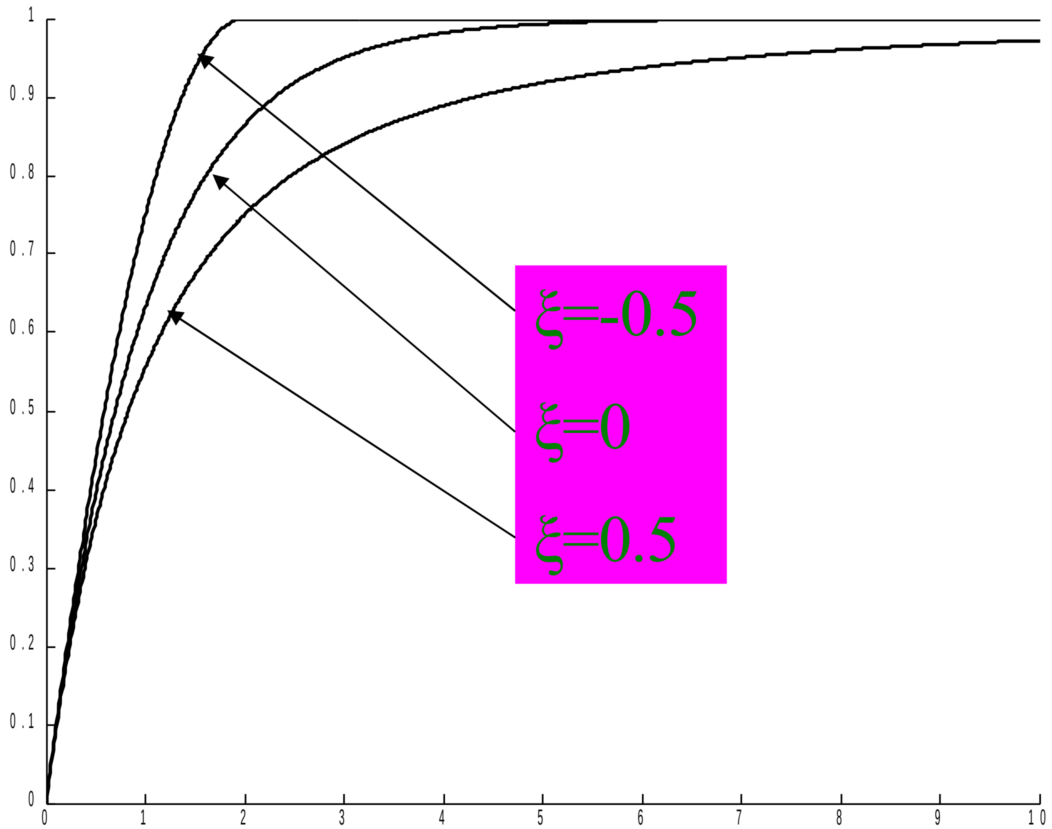
Limit theorem 2: The distribution of POT values approaches the following distribution function for growing threshold

$$G(x) = \begin{cases} 1 - \left(1 + \xi \dfrac{x}{\sigma}\right)^{-\frac{1}{\xi}} & \text{für} \quad \xi \neq 0 \\ 1 - e^{-\frac{x}{\sigma}} & \text{für} \quad \xi = 0 \end{cases}$$

Pickands 1975, generalized Pareto distribution

Extreme value theory

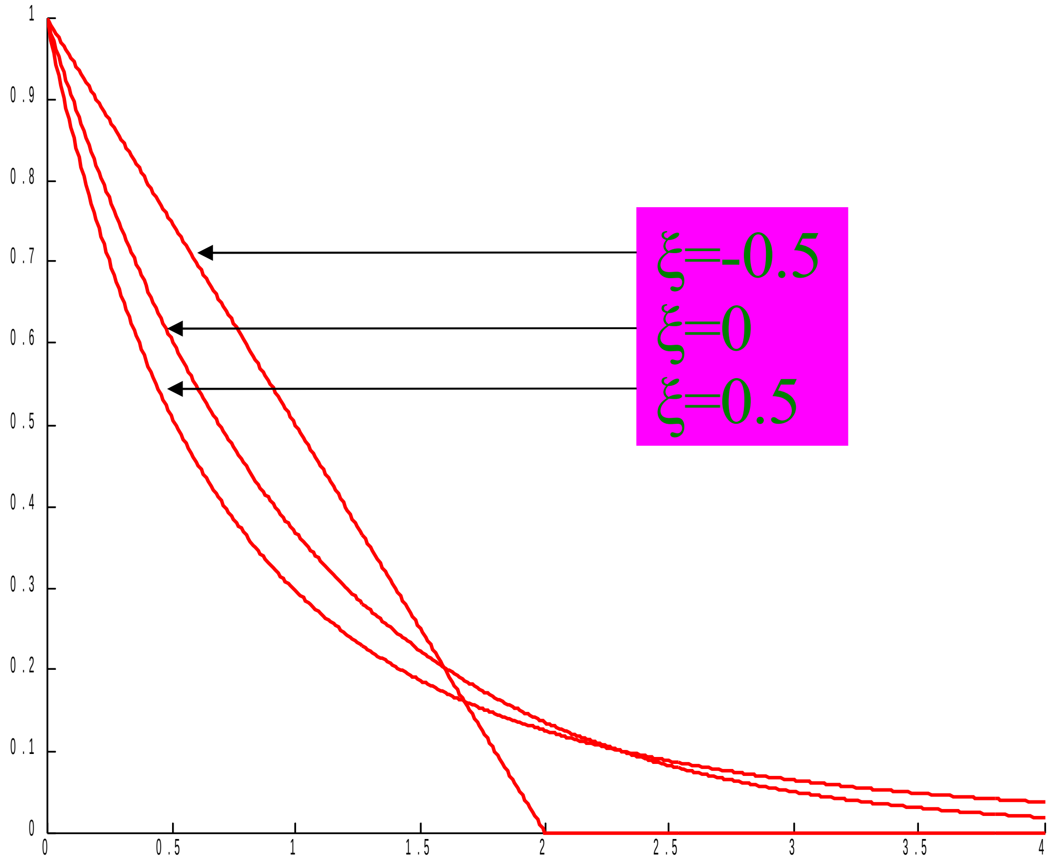
Distribution function G(x)



σ=1

Extreme value theory

probability density g(x)



σ=1

Wind and loads

Mechanical components

Gravitation, Inertia

deterministic!

Aerodynamical components

induced by turbulence

stochastic!

Loads in the simulation

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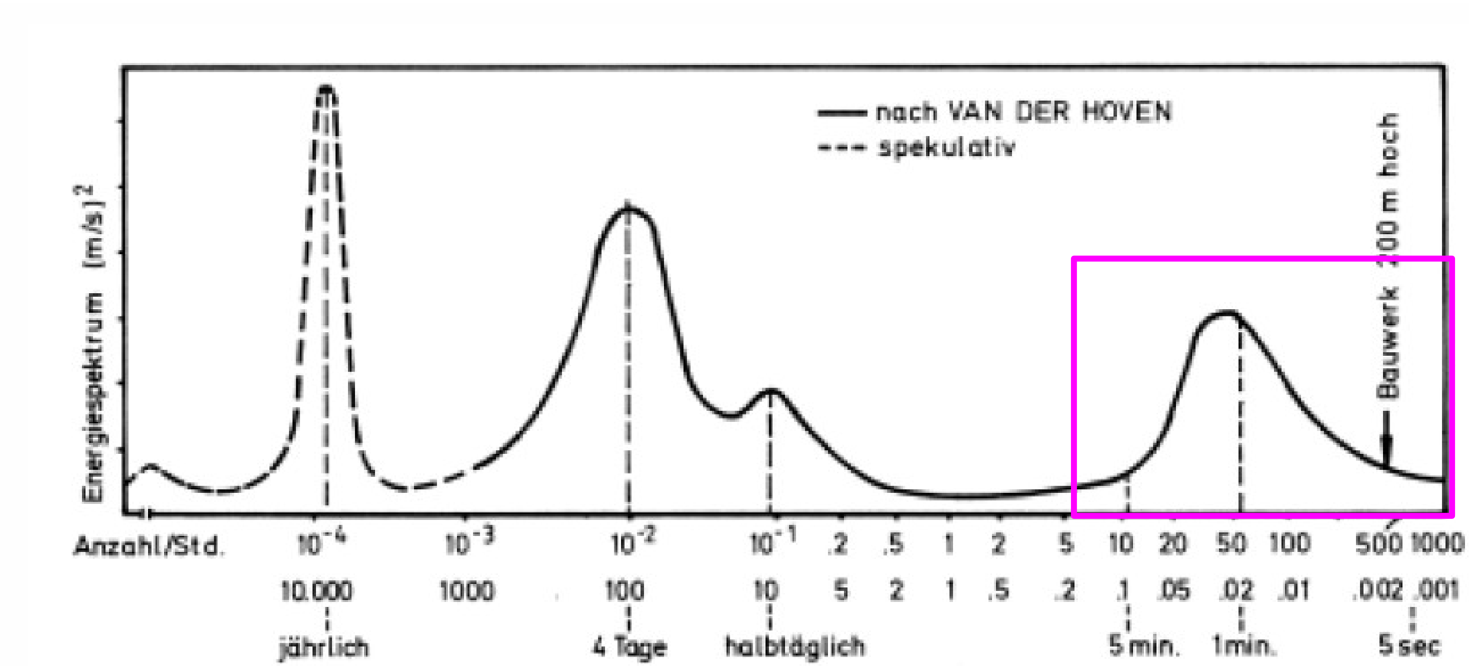
Wind and loads

Wind as stochastic process

different time scales: diurnal change

seasons

turbulence



Wind and Loads

Wind in the simulation

- classification by 10 min. means
- coverage of low frequency influences due to frequency distributions (Weibull, Rayleigh)
- coverage of turbulence due to specification of energy spectra (von Karman, Kaimal)
- coverage of extreme events due to specification of corresponding velocities (v_{50}, v_1)

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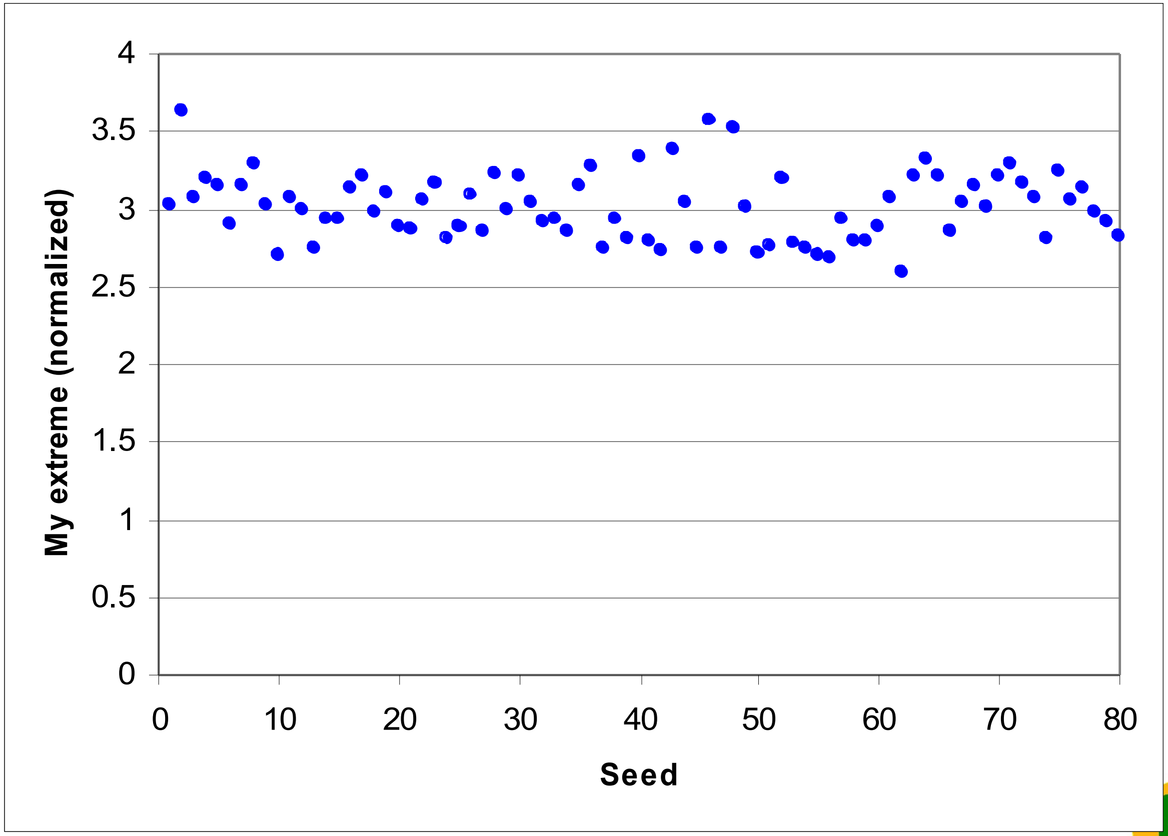
Example

Extreme values from 80 time series of 10 min.



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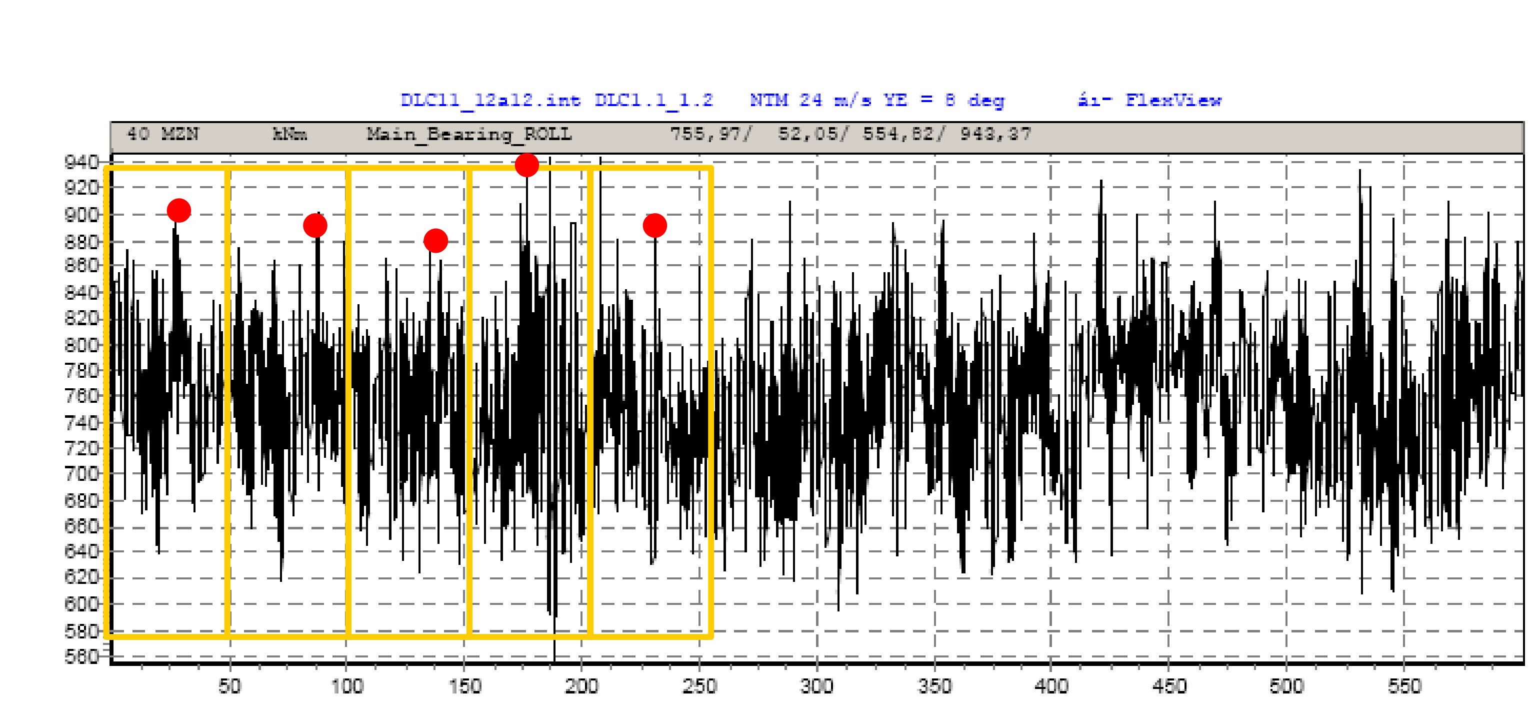
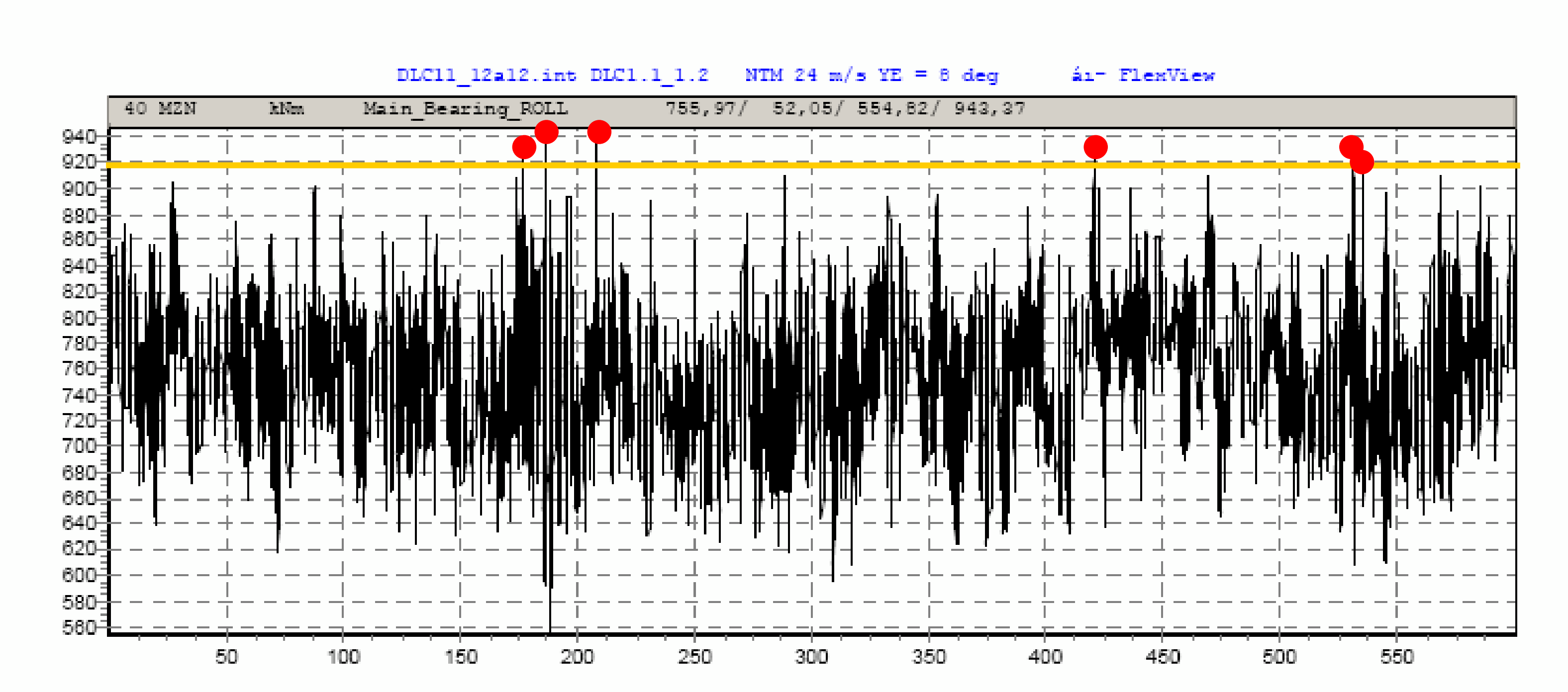
Example

Method of block extremes (classical method, Gumbel)

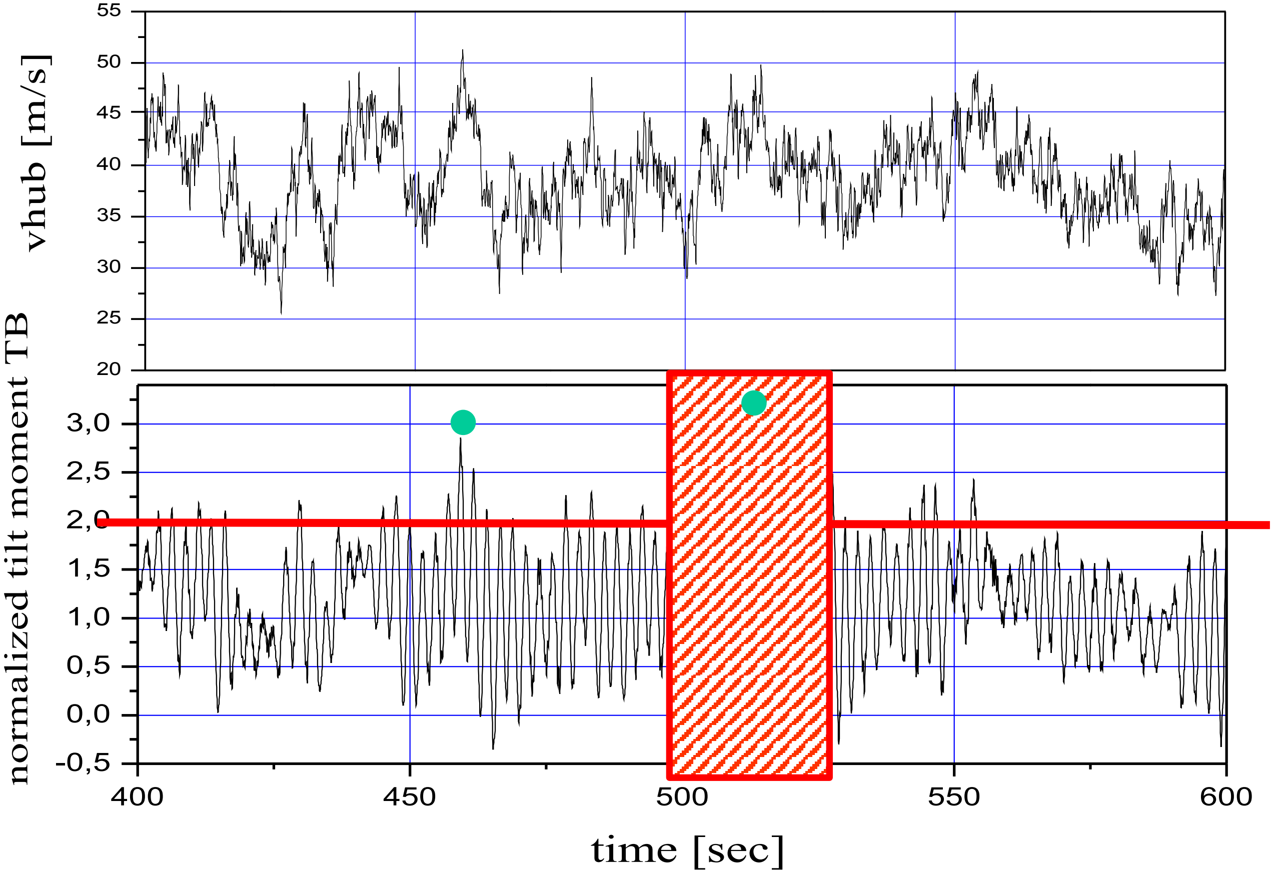


Example

POT-method (peak over threshhold) Pickands



Example



Events have to be statistically independent!

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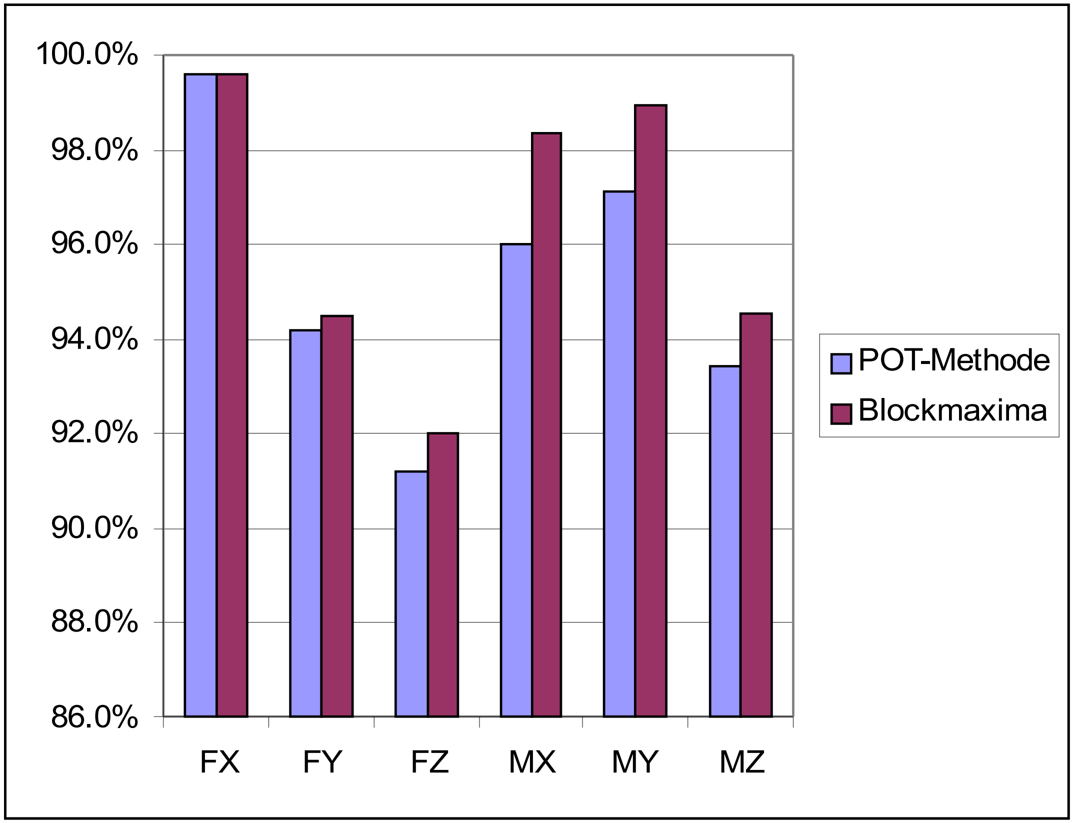
Example

Comparison of both methods – 50-years values



System Tb

relativly to the observed extreme

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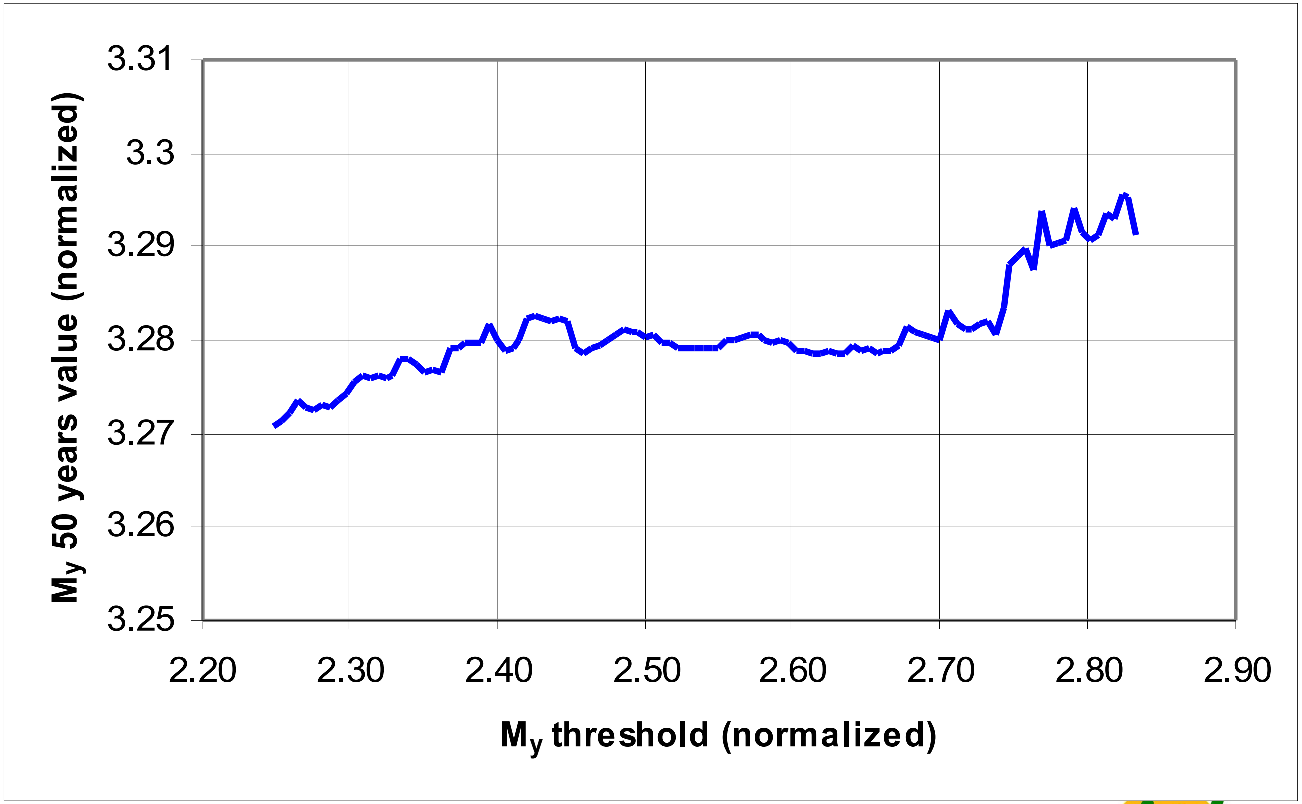
Example

Dependence on threshold



Gumbelwind

Computer program „*Gumbelwind*“

for the calculation of ultimate load of wind energy converters

Program flow:

- choose threshold and apply POT-method
- fitting of Pareto distribution (maximum likelihood)
- quality checks: χ^2 test of goodness of fit, error estimates
- calculation of the 50-years value with

$$P(M_{y,ext},T) = \sum_{v} [1 - (P^{v}_{\sigma,\xi}(M_{y,ext}))^{n}] \cdot p(v) = 3.8 \cdot 10^{-7}$$

Gumbelwind

evaluation for system: N

observed Absmax-values and simultaneous loads

	FX	FY	FZ	MX	MY	MZ
FX	247.2	-10.7	170.7	-378.8	-84.6	733.8
FY	218.4	65.3	349.2	624.3	-564.3	682.7
FZ	215.7	17.0	529.7	570.3	-211.5	961.2
MX	221.1	-9.5	191.7	-1071.5	-218.3	779.8
MY	214.6	39.5	442.1	513.8	-1432.1	287.8
MZ	200.3	2.4	453.9	-33.5	95.7	1203.7

classical Absmax-evaluation of the data files

50 years extremes and simultaneous loads according to extreme value theory

	FX	FY	FZ	MX	MY	MZ
FX	267.5	-14.4	239.6	-1097.6	-472.7	957.0
FY	224.5	55.2	361.2	645.5	-1090.0	777.0
FZ	226.3	32.8	466.1	541.7	-539.7	914.1
MX	230.4	-28.0	238.8	-1189.1	-569.3	918.8
MY	224.3	20.5	217.4	556.3	-1476.3	883.1
MZ	226.9	24.5	419.2	-348.1	350.2	1087.1

calculation of 50-years values with the help of extreme value theory

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Gumbelwind

MY-distribution parameters for single wind bins

V_hub	V_dir	Turb-int	threshold	count	shape	scale	ratio	rel_err	chi^2
m/s	°	%	kN				%	%	
4.0	0.0	0.34	-529.6	14.6	-0.143	64.6	0.0	0.9	1
6.0	0.0	0.27	-545.9	14.6	-0.355	79.0	0.0	2.0	1
8.0	0.0	0.23	-561.1	14.6	-0.338	88.3	0.0	1.9	1
10.0	0.0	0.21	-582.5	14.6	-0.279	96.4	0.0	1.8	1
12.0	0.0	0.19	-651.4	14.6	-0.272	101.3	0.0	2.1	1
13.0	0.0	0.19	-673.1	14.6	-0.386	143.2	0.0	2.5	1
14.0	0.0	0.18	-714.2	14.6	-0.351	132.7	0.0	2.4	1
16.0	0.0	0.18	-779.0	14.6	-0.153	119.4	99.2	2.3	1
18.0	0.0	0.17	-839.8	12.1	-0.250	120.6	0.0	1.8	1
20.0	0.0	0.16	-845.3	14.3	-0.216	133.1	0.0	3.6	1
22.0	0.0	0.16	-849.2	14.6	-0.337	159.7	0.0	1.3	1
24.0	0.0	0.16	-878.0	14.6	-0.332	160.9	0.0	1.9	1
25.0	0.0	0.16	-879.8	14.6	-0.377	169.1	0.0	1.5	1
40.0	0.0	0.11	-407.9	14.6	-0.115	55.2	0.0	2.7	1
45.0	0.0	0.11	-432.2	14.6	-0.194	70.2	0.0	1.1	1
47.5	0.0	0.11	-483.2	10.3	-0.194	68.2	0.0	3.3	1
50.0	22.0	0.11	-771.4	14.6	-0.152	158.9	0.8	2.4	1
50.0	0.0	0.11	-486.7	14.6	-0.081	72.1	0.0	1.5	1

wind related quantities

parameters of the extreme value distribution

qualitative quantities